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(H.W) i) $(p \Rightarrow q) \Leftrightarrow (\sim p \vee q)$

ii) $\sim(p \Rightarrow q) \equiv \{p \wedge (\sim q)\}$

iii) $[(p \wedge q) \Rightarrow p] = [q \wedge \sim p]$

Sol. i) Truth table of $(p \Rightarrow q) \Leftrightarrow (\sim p \vee q)$

p	q	$\sim p$	$p \Rightarrow q$	$\sim p \vee q$	$(p \Rightarrow q) \Leftrightarrow (\sim p \vee q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

ii) Truth table of $\sim(p \Rightarrow q) \equiv \{p \wedge (\sim q)\}$

p	q	$p \Rightarrow q$	$\sim(p \Rightarrow q)$ A	$\sim q$	$p \wedge (\sim q)$ B	$A \Leftrightarrow B$
T	T	T	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	F	F	T
F	F	T	F	T	F	T

iii) Truth table of $(p \wedge q) \Rightarrow p = [q \wedge \sim q]$

p	q	$p \wedge q$	$(p \wedge q) \Rightarrow p$ (A)	$\sim q$	$q \wedge \sim q$ (B)	$A \Leftrightarrow B$
T	T	T	T	F	F	F
T	F	F	T	T	F	F
F	T	F	T	F	F	F
F	F	F	T	T	F	F

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5) De - Morgan's laws :-

i) $\sim(p \vee q) \Leftrightarrow (\sim p) \wedge (\sim q)$

ii) $\sim(p \wedge q) \Leftrightarrow (\sim p) \vee (\sim q)$

i)

p	q	p∨q	~(p∨q) (A)	~p	~q	(~p)∧(~q) B	A⟺B
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T

it is tautology

ii)

p	q	p∧q	~(p∧q) (A)	~p	~q	(~p)∨(~q) (B)	A⟺B
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

6) Ident...

i) $p \wedge t$

ii) $p \vee f$

i) :-

p
T
F

ii) :-

p
T
F

7) Comple...

i) $p \vee ($...

ii) $p \wedge ($...

i)

p	~
T	F
F	T

ii) $p \wedge ($...

p
T
F

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6) Idenitity Laws :-

i) $p \wedge t = t \wedge p = p$
ii) $p \vee f = f \vee p = p$

i) ⇒

p	t	$p \wedge t$	$t \wedge p$
T	T	T	T
F	T	F	F

ii) ⇒

p	f	$p \vee f$	$f \vee p$
T	F	T	T
F	F	F	F

7) Complement Laws :-

i) $p \vee (\sim p) = t$
ii) $p \wedge (\sim p) = f$

i)

p	$\sim p$	$p \vee (\sim p$	t
T	F	T	T
F	T	T	T

ii) $p \wedge (\sim p) = f$

p	$\sim p$	$p \wedge (\sim p)$	f
T	F	F	F
F	T	F	F

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H.W i) $(P \Rightarrow q \wedge r) \vee (\sim P \wedge q)$

HW.

Ex 5)

P	q	r	$q \wedge r$	$P \Rightarrow (q \wedge r)$	$\sim P$	$\sim P \wedge q$	$(P \Rightarrow q \wedge r) \vee (\sim P \wedge q)$
T	T	T	T	T	F	F	T
T	T	F	F	F	F	F	**F**
T	F	T	F	F	F	F	**F**
T	F	F	F	F	F	F	**F**
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

ii) $(P \vee q \Rightarrow S) \Leftrightarrow (P \Rightarrow S) \vee (q \Rightarrow S)$

P	q	S	$P \vee q$	$P \vee q \Rightarrow S$ (A)	$P \Rightarrow S$	$q \Rightarrow S$	$(P \Rightarrow S) \vee (q \Rightarrow S)$ (B)	$A \Leftrightarrow B$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	**T**
T	F	T	T	T	T	T	T	T
T	F	F	T	F	F	T	T	F
F	T	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T	F
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

Ex. 13 (1)

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H.W.

Ex5) The truth value of $P \Leftrightarrow q$ is F.

$P \Leftrightarrow q$	p	q	$p \vee q$
F	T	F	T
	F	T	T

The truth value of $P \Leftrightarrow q$ is T

$P \Leftrightarrow q$	p	q	$p \vee q$
T	T	T	T
	F	F	F

Ex. 13:-1) $(p \Leftrightarrow q) \wedge (r \vee q)$.

p	q	r	$P \Leftrightarrow q$	$r \vee q$	$(P \Leftrightarrow q) \wedge (r \vee q)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	F	F	F
F	T	T	F	T	T
F	T	F	F	T	F
F	F	T	T	T	T
F	F	F	T	F	F

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ii) $(p \vee q) \wedge (\sim r) \Rightarrow q$

p	q	∼r	p∨q	(p∨q)∧(∼r)	(p∨q)∧(∼r)⇒q
T	T	F	T	F	T
T	T	T	T	T	T
T	F	F	T	F	F
T	F	T	T	T	F
F	T	F	T	F	T
F	T	T	T	T	T
F	F	F	F	F	T
F	F	T	F	F	T

H.W. i)

Q ⇒ $(\sim q \Rightarrow \sim p) \wedge (q \Rightarrow p) \Rightarrow (p \Leftrightarrow q)$

p	q	∼p	∼q	(∼q⇒∼p) (A)	(q⇒p) (B)	A∧B (C)	p⇔q (D)	C⇒D
T	T	F	F	T	T	T	T	T
T	F	F	T	F	T	F	F	T
F	T	T	F	T	F	F	F	T
F	F	T	T	T	T	T	T	T

iii) $(p \Leftrightarrow q \wedge r$

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

H.W. ⇒

Ex 10) a) $(p \Rightarrow q$

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

iii) $(p \Leftrightarrow q \wedge r) \Rightarrow (\sim r \Rightarrow \sim p)$.

p	q	r	$(p \Leftrightarrow q)$ (A)	$(A \wedge r)$ C	$\sim r$	$\sim p$	$\sim r \Rightarrow \sim p$ (B)	$B \Rightarrow C$
T	T	T	T	T	F	F	T	T
T	T	F	T	F	T	F	F	T
T	F	T	F	F	F	F	T	F
T	F	F	F	F	T	F	F	
F	T	T	F	F	F	T	T	
F	T	F	F	F	T	T	T	
F	F	T	T	T	F	T	T	
F	F	F	T	F	T	T	T	

H.W. ⇒

Sec 10) a) $(p \Rightarrow q \wedge r) \vee (\sim p \wedge q)$

p	q	r	$q \wedge r$	$p \Rightarrow q \wedge r$ (A)	$\sim p$	$\sim p \wedge q$ (B)	$A \vee B$
T	T	T	T	T	F	F	T
T	T	F	F	F	F	F	F
T	F	T	F	F	F	F	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

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Ex 1(c) b) i) $(p \vee q \Rightarrow s) \Leftrightarrow (p \Rightarrow s) \vee (q \Rightarrow s)$

p	q	s	$p \vee q$	$p \vee q \Rightarrow s$ (A)	$p \Rightarrow s$	$q \Rightarrow s$	$p \Rightarrow s \vee (q \Rightarrow s)$ (B)	$A \Leftrightarrow B$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	F	T	T	F
F	T	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T	F
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

ii) $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow [p \Rightarrow r]$

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$ (A)	$p \Rightarrow r$ (B)	$A \Rightarrow B$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$\mathbb{C} \vee P = P$

$\mathbb{C} \wedge P = \mathbb{C} \to$ null (Contradiction)

Identity $\wedge P = P$ | $I \wedge P = P$

$I \vee P = I$ | $I \vee P = I$

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15) Symplify the following :-

i) $(P \wedge Q) \vee P$

Solution: $(P \wedge Q) \vee P \Rightarrow [(P \wedge Q) \vee P \equiv P \vee (P \wedge Q)]$

$\Rightarrow P \vee (P \wedge Q) \quad [P \wedge T \equiv P]$

$\Rightarrow (P \wedge T) \vee (P \wedge Q)$ (Distributive)

$\Rightarrow P \wedge (T \vee Q)$

$= P \wedge T$

ii) $(P \wedge Q) \wedge \sim P \Rightarrow$ ~~$\sim P \vee$~~

$= (P \wedge Q) \wedge \sim P = \sim P \wedge (P \wedge Q)$

$= (\sim P \wedge P) \wedge (\sim P \wedge Q)$ (By Distributed)

$= \mathbb{C} \wedge (\sim P \wedge Q)$

where $\mathbb{C}$ is contradiction

$= \mathbb{C}$

Varification by truth table:

P	Q	P∧Q	~P	(P∧Q) ∧ ~P
T	T	T	F	F
T	F	F	F	F
F	T	F	T	F
F	F	F	T	F

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v) $\sim(\sim P \wedge Q) \wedge (\sim P \vee Q) \wedge (P \vee Q)$

H.W. i) $[P \wedge (P \Rightarrow a)] \Rightarrow q$

P	q	a	$P \Rightarrow a$	$P \wedge (P \Rightarrow a)$	$[P \wedge (P \Rightarrow a)] \Rightarrow q$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	F	T
F	T	T	T	F	T
F	T	F	T	F	T
F	F	T	T	F	T
F	F	F	T	F	T

ii) $[(P \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow [P \Rightarrow r]$

P	q	r	$P \Rightarrow q$	$q \Rightarrow r$	$(P \Rightarrow q) \wedge (q \Rightarrow r)$ (A)	$P \Rightarrow r$ (B)	$A \Rightarrow B$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

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CHAPTER ⇒ 2
:- QUANTIFIERS :-

1) $\forall$. forall , forevery [universal quantifiers]

2) $\exists$ ⇒ forsome , atleast [There exist]

Example:- The statement $(\forall n \in N)\ (n+2>1)$ is true since its truth set.

$$T(P) = \{n : n \in N ;\ n+2>1\}$$
$$= \{1, 2, 3, 4 \ldots\} = N$$

2) The statement $(\forall x \in R)\ (x^2 > -1)$ where R is the set of real numbers, is true since its truth set.

$$T(P) = \{x : x \in R ;\ x^2 > -1\} = R$$

B

$\exists$ Examples:

1) The statement $(\exists n \in N)\ (n+7<6)$ is false since

$$T(P) = \{n : n+7<6\} = \phi.$$

* Negation of a Quantifier

Ex 1:-) The square of every real number
⇒ $\forall x\ [R(x) \Rightarrow \sim G(0, x^2)]$.

ix) ⇒ $\forall x \sim (x^2 = 1$ and $x^2 - 2x + 3 = 0)$
⇒ $\forall x\ (x^2 \neq 1$ and $x^2 - 2x + 3 \neq 0)$

Chapter :- 2

Quantifiers :-

Introduction :-

The identity elements for the operations $\cup$ and $\cap$ are respectively empty set ϕ and universal set I for operation $\vee$ and $\wedge$ are 0 (flase F) and 1 (True T) and for operations of (+) and (·) are 0 and 1.

1. UNIVERSAL Quantifier :-

The symbol $\forall$ which read as 'for every' or 'for all' is called the universal quantifier.

Ex :- 1 The statement $(\forall n \in N)(n+2 > 1)$ is true since its truth set

$$\Rightarrow \quad T(P) = \{n : n \in N, n+2 > 1\}$$
$$= \{1, 2, 3, 4 \ldots\} = N.$$

Ex :- 2 The statement $(\forall n, \in N)(n+1 > 5)$ is false since truth set

$$\Rightarrow \quad T(P) = \{5, 6, 7, \ldots\ldots\} \neq N$$

2. Existential quantifiers :-

The symbol $\exists$ which is read as 'there exists' or 'for at least one' is called the existential quantifier.

Ex:-1 The statement $(\exists n \in N)(n+7<6)$ is false since

$\Rightarrow$ $$T(P) = \{n : n+7<6\} = \phi$$

Ex:-2 The statement $(\exists x \in R)(x^2-1=0)$ R is the set of real number, is true since

$\Rightarrow$ $$T(P) = \{x : x^2-1=0\} = \{1, -1\} \neq 0$$

3. Negation of a quantifier :-

consider the following proposition. 'All Indians are honest'. If M denotes the set of Indians, then the above in symbolic form is

$$(\forall x \in M)\ (x \text{ is honest})$$

The above statement will become false, if we say that 'there is an Indian who is not honest' or in Symbolic form

$$(\exists x \in M)\ (x \text{ is not honest})$$

~~· The square of every real number~~

$\Rightarrow \cancel{\forall x [R(x \Rightarrow \sim G(0, x^2)]}$

4. De-Morgan Laws:- $\sim(\forall x \in D) P(x) \equiv (\exists x \in D) \sim P(x)$

$\sim(\exists x \in D) P(x) \equiv (\forall x \in D) \sim P(x)$

Ex:- 1

Translate the following sentence into symbols:

(i) 4/5 is a rational number

$\Rightarrow Q(4/5)$

(ii) $\sqrt{3}$ is a irrational number

$\Rightarrow \sim Q(\sqrt{3})$

(iii) Every rational number is a real number

$\Rightarrow \forall x [Q(x) \Rightarrow R(x)]$

(iv) Some real number are rational

$\Rightarrow \exists x [R(x) \wedge Q(x)]$

(v) The square of every real number is not number

$\Rightarrow \forall x\,[R(x) \Rightarrow \sim G(0, x^2)]$

Ex :- 2

Negate each of the following statement :

(i) $\exists x\,(x^2 = 1$ and $x^2 - 2x + 3 = 0)$

$\Rightarrow \forall x \sim (x^2 = 1$ and $x^2 - 2x + 3 = 0)$

$\forall x\,(x^2 \neq 1$ or $x^2 - 2x + 3 \neq 0)$

(ii) $\exists x\,(x + 2 = x)$

$\Rightarrow \forall x\,(x + 2 \neq x)$